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DISCUSSION OF PROCEEDINGS - SEPARATES

354, 361, 362, 366, 390, 413, 431, 432

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DISCUSSION OF HYDRAULIC FUNDAMENTALS OF CLOSED CONDUIT SPILLWAYS PROCEEDINGS-SEPARATE NO. 354

GEARY M. ALLEN, Jr.,[1] J.M. ASCE.—The writer agrees that the phenomenon of flow in a closed conduit system presented by the author may appear novel to many engineers; however, virtually the same flow condition has been confronted by hudraulic engineers engaged in the design of flood control outlets through large dams for many years.

Of necessity, conduits constructed through the upper levels of masonry dams are laid on a slope which becomes tangent to the face of the overflow section at the conduit's exit. In most cases this design requirement results in a conduit laid on a steep slope which could produce a flow condition through the conduit similar to that which the author describes.

Of particular importance to the designer is the location of the hydraulic control of the steep conduit with respect to reservoir stage. Generally, the location of the control with respect to reservoir stage is estimated by assuming the control at the outlet end of the conduit (full flow condition), computing the rating curve of the conduit for this condition and comparing this curve with the part full rating curve.

The pressure or hydraulic grade line throughout the length of the steep conduit and especially immediately below the operating gates is of significance because as the author states, if the friction grade line falls below the conduit (conduit centerline in the case of fig 6a) pressures less than atmospheric will occur with resultant cavitation tendencies. It must be remembered that the hydraulic grade line represents the average pressures and not the maximum or minimum pressures that may occur, consequently, care should be exercised in the selection of allowable average pressure reductions.

The author is to be complimented on his description of the actual phenomenon, and on his presentation of a more general method for the determination of the pressures within the conduit.

WEN-HSIUNG LI,[2] A.M. ASCE.—The phenomenon of pipes flowing full at steep slopes regardless of the outlet condition has been observed independently by the writer and by other investigators[3] in connection with studies of flow through culvert-pipes with rounded entrance. It is of great interest to the writer that this phenomenon has also been observed by the author on pipes with sharp-cornered entrance.

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3. Straub, L. G., Anderson, A.G. and Bowers, C.E., "Importance of Inlet Design on Culvert Capacity," Research Report No. 15-B, Highway Research Board, 1953.

The advantage of having the pipe flowing full at steep slopes can be brought out forcefully by a graph similar to Figure a. This graph has been plotted for the determination of culvert-pipe size for known values of allowable depth of ponding h above the invert at the entrance, the discharge Q, the slope of the invert s, and the Manning's roughness coefficient n of the pipe. When $\frac{sh^{1/3}}{n^2}$ is greater than about 100 and $\frac{D}{h}$ is from approximately 0.65 to 0.87, the flow in the pipe is pulsating between full flow and part-full flow. The values of $\frac{Q}{h^{5/2}}$ shown in the graph are the minimum values observed. For pipes flowing full at slope s other than the friction slope, the values of $\frac{Q}{h^{5/2}}$ depend also on the length of the pipe. The values shown in the graph serve to indicate only the general trend of variation. As can be seen from the graph, larger pipe does not necessarily give larger discharge with given values of h, s and n. Under certain conditions, the same capacity can be obtained with pipes greatly different in size. For example, if the design conditions are such that $\frac{sh^{1/3}}{n^2} = 800$ and $\frac{Q}{h^{5/2}} = 3.0$, a pipe with $D = 1.3\,h$ or a pipe with $D = 0.6\,h$ may be used. The larger pipe will flow partly full while the smaller pipe will flow full with subatmospheric pressure in the barrel. Pipe-culverts with sharp-cornered entrance have not been observed to flow full under practical conditions. For this case, the capacity always decreases with the pipe size, and in the example above, the larger pipe must be used. Obviously, it is of great practical interest to find the general criteria for maintaining full flow in a closed conduit at steep slope.

In studies of the hydraulics of closed conduits, the hydraulic grade line is generally plotted at a distance above the center line of the conduit equal to the pressure head at the center line. This method is correct only when the pressure is hydrostatically distributed across the cross-section. At the free outlet of a pipe flowing full, the pressure is not hydrostatically distributed due to the slight curvature of the surfaces of the jet. However, the pressure is hydrostatically distributed at a very short distance upstream from the outlet. At this point, the pressure head at the center line is equal to D/2. Thus the hydraulic grade line should meet the crown of the conduit at the outlet as shown in Figure 6. The available head across the structure is thus equal to the difference in elevation between the free-water surface above the entrance and the crown of the conduit at the free outlet. Thus the value of z in Eq. 1 should be equal to the difference in elevation between the crest of the weir and the crown of the conduit at the free outlet. This correction may be very small for the example given by the author. However, this correction of D/2 in the available head may be extremely important when D/2 is not small compared with the available head, as in the case of culverts for highway drainage.

The author has mentioned model testing to determine whether cavitation would occur in a given structure. It is of practical interest to have a method of predicting the occurrence of cavitation without resort to

model tests. The minimum pressure occurs at the vena contracta near the entrance. Since the loss of head upstream of this section is negligible, the minimum pressure can be easily shown to be (see Figure b)

$$\frac{p_{min.}}{w} = (1 + K_e - \alpha)\frac{v_o^2}{2g} - (s - s_f)L$$

If c is used to denote the coefficient of contraction at the vena contracta, α can be shown to be $\frac{1}{c^2}$. This equation awaits verification by observation.

ARTHUR L. COLLINS,[1] A.M. ASCE.—In the Separate 354, and with reference to the "CONCLUSIONS", second line, the word "may" could imply that the conduit could flow full or partially full without submergence. The writer prefers to argue that the conduit may not safely maintain a high efficiency unless it is submerged, and offers the siphon outlet as an example.

In the northern California delta, between Sacramento and Stockton, there are about 500,000 acres of reclaimed submerged tidal lands. Large levees surround man-made islands with areas of a few hundred to several thousand acres. First the levee is constructed from material borrowed from outside the area, thus leaving a deep channel. Inside the area the excess rain and seepage is brought to a suitable location by a large drainage canal, where it is pumped over the levee.

A typical modern pump installation, of which there are perhaps 100 with discharge pipes of 18 to 30 inches, consists of vertical pump installed at the terminal of the drain canal at the levee. A total of 200 or 300 feet of pipe extends from the pump to the base of the levee, thence up the levee slope, thence nearly level through the levee where it again descends until the end can be submerged in the waste canal.

The crest of the siphon will be about 20 feet above the surface in the drainage canal, and 15 feet higher than the waste canal, thus leaving a net lift of about 5 feet for the pump. It is a simple matter to anticipate the hydraulic gradient in the pipe system and the head available from the pump. However, water gas always forms from the high rotation of the pump impeller, and accumulates in the crest of the siphon. Also, a reduction of the pressure at the siphon crest causes the gas to expand. It eventually stops the operation of the siphon unless the system is so designed that the gas capsule is broken into small volumes and flows down the pipe with the water. Expensive mechanical installations have been provided in some cases to take care of this trouble, when the proper tilt to the pipe and a deflector in the pipe would have solved the problem in an inexpensive and simple way.

In the same area there are hundreds of smaller sizes of pipes, 4 inches to 12 inches in diameter, which are true gravity siphons, and which siphon the water from the waste canal back over the levee for irrigation purposes. In observing these gravity siphons and pump siphons over a series of years, one will find conditions such as pipe ends rusted off, or extra low water, which will expose the pipe ends. When

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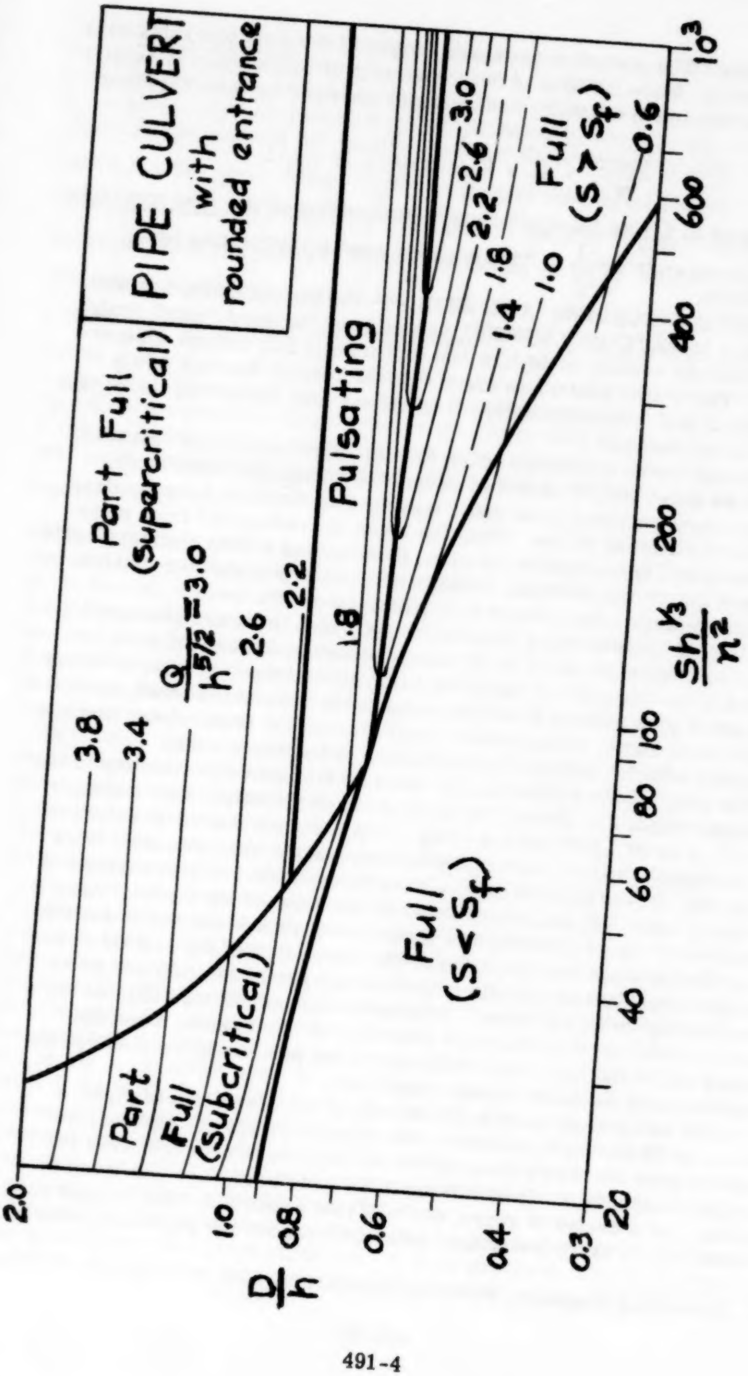
PIPE CULVERT
with
rounded entrance
Part Full
(Supercritical)
Pulsating
Part
Full
(Subcritical)
Full
(S < S_f)
Full
(S > S_f)
Q/h^{5/2} = 3.0
3.8
3.4
2.6
2.2
1.8
3.0
2.6
2.2
1.8
1.4
1.0
0.6
D/h
2.0
1.0
0.8
0.6
0.4
0.3
20
40
60
80
100
200
400
600
10^3
Sh^{1/3}/n^2

Fig. a

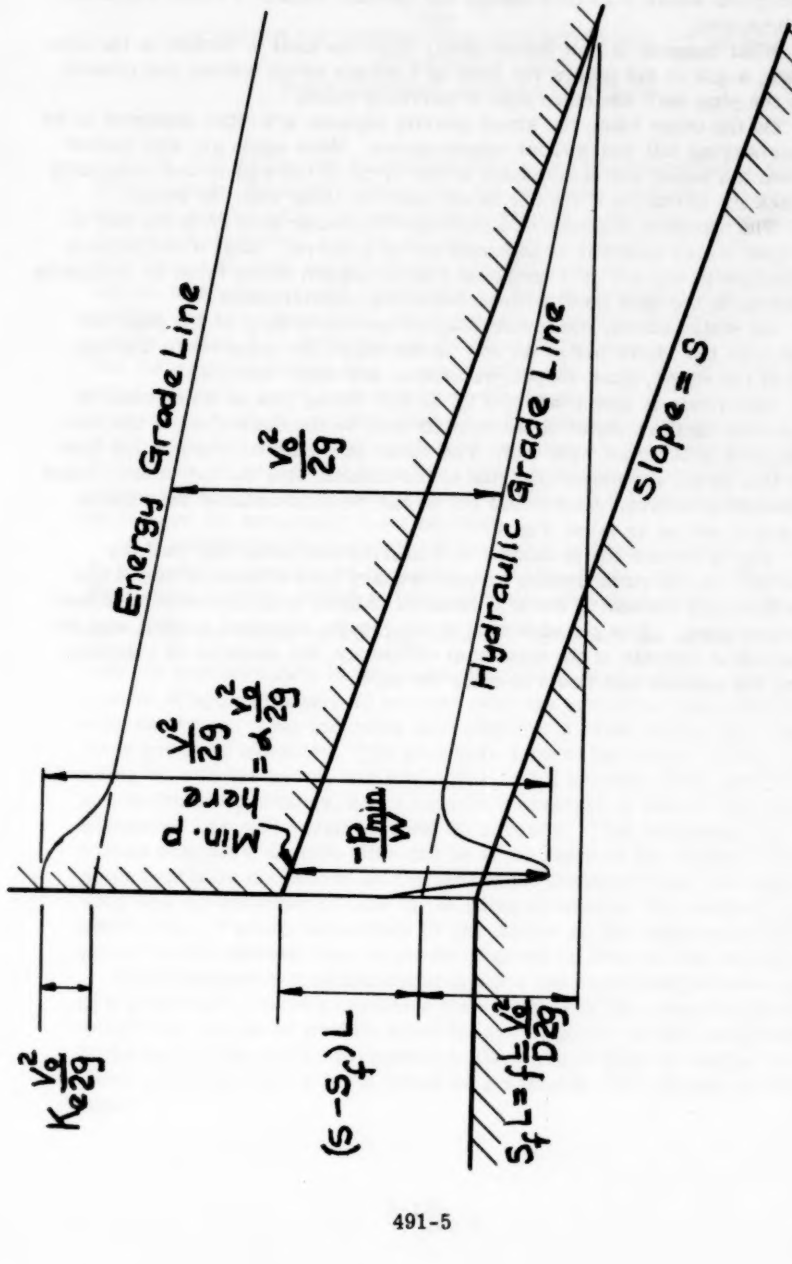
Energy Grade Line
Hydraulic Grade Line
Slope = S
$\frac{V_o^2}{2g}$
$\frac{V^2}{2g} = \alpha \frac{V_o^2}{2g}$
Min. p here
$-\frac{P_{min}}{w}$
$K_e \frac{V_o^2}{2g}$
$(S-S_f)L$
$S_f L = f \frac{L}{D} \frac{V_o^2}{2g}$

Fig. b

this occurs there is but one chance in a hundred that the pump siphons will not break and cause an enormous overload on the pump motor, due to an increase in the lift of from 6 or 7 feet to 20 feet. There was one exception where a 20 inch siphon did operate without a break when not submerged.

What happens is that immediately after the seal is broken at the pipe edge, a gob of air enters the pipe as a wedge which widens and travels up the pipe until the down pipe is partially filled.

On the other hand, the small gravity siphons are often observed to be discharging full and without submergence. Here again air will collect from the water and accumulate at the crest of the siphon and eventually block its operation if the air is not carried along with the water.

The simplest example of a full pipe discharge is seen in the use of a hose where gasoline is siphoned out of a barrel. Also it is common practice to use a 2 or 3 inch bent pipe to siphon water from an irrigation lateral to the land level without requiring submergence.

As stated above, the break away or hydraulic drop at the pipe end permits the air to wedge its way up the pipe. Its occurrence depends upon the shape, size, slopes, velocities and other factors.

Reference is now made to Fig. 1, (b). Using this as an example it appears that the outlet of the conduit may be the equivalent of the outfall of a submerged pipe end. The water in the outlet basin is not free to fall away, and expose the end of the conduit, and the air wedge cannot become effective. Such would not be the condition should the pipe or conduit end as in (a) of Fig. 2.

Can it be that the prototype on a small scale tells only half the story? In the pump siphon an unnecessary loss of head of only a few inches may amount of several hundred dollars in an increase of annual power cost. It is possible that if the gravity operated conduit was required to operate at its maximum efficiency, the practice of submerging the conduit end would become the rule.

DISCUSSION OF PRESSURE SURGE CONTROL AT TRACY PUMPING PLANT PROCEEDINGS-SEPARATE NO. 361

C. E. WITHERS.—In discussing the pump characteristics of the Tracy pumps the author states that the pump performance data necessary for investigating the transient hydraulic conditions in the zone of "energy dissipation" and the zone of "turbine operation" were not available from the pump manufacturer and had to be estimated from the curves of another double volute pump. Due to the expense involved in making the necessary tests, no pump manufacturer will have this information unless it is required and paid for by the purchaser. The lack of this information means that in most cases when investigating the transient hydraulic conditions in a pump discharge system, the pump characteristics in these regions must be estimated from the known curves of the very few pumps that have been run through the necessary tests. It would be of interest to have the author's opinion on whether the error induced in making such assumptions could be of sufficient consequence to require the necessary tests be made.

It is interesting to compare the transient hydraulic conditions in the Tracy pumping system as designed, with the system without any valves. Neglecting friction, which in this case will not induce appreciable error, and using the values of 2ρ, K_I and $\frac{2L}{a}$ given by the author, we find that the pressure rise at the pump would have been $1.40\,H_R$ or an increase of approximately 20 percent over the pressure rise obtained with the valve. This indicates how effective a valve of this type can be when properly operated. The pressure drop at the pump will be the same as with the valve, and will occur in 3.5 seconds after power failure to the pump motors, and a complete reversal of flow in the pump discharge line will occur in about 10 seconds. This indicates that valves with slow closure time can be of no value in the control of transient hydraulic conditions as the maximum pressure rise, the maximum drop and the reversal of flow in the pump discharge line occur in a very short time. Further comments by the author on his experience in the proper valve closure time would be of great interest to this writer.

Quite frequently it is desirable to know the maximum reverse speed of a pump and to have a complete time history of the transient hydraulic conditions in a pump system after failure of power to the pump motors. If the author has such information in the form of curves similar to those shown in Figure 14, it would be a valuable contribution to this paper.

P. LINTON.[1] —The author has performed a most valuable service in reporting these full-scale tests on pressure surges after power failure, information which is unfortunately published only in very few cases. They show up clearly the value of a full analysis of transient conditions in leading to savings in surge control devices such as surge tanks and control valves.

The time history of the pressure surge, Fig. 3, is most interesting and brings out clearly the results of the pressure surge diagrams, Figs. 11 and 12. Comparison between the measured and computed curve shows good agreement as far as the general shape is concerned; the lowest pressure reached is some 17 ft higher than the calculated one, while the subsequent pressure rise is 26 ft less than that computed. In both cases the measured pressures are more favourable than the calculated ones and the present tests therefore confirm that a careful analysis of the pressure surges leads to a somewhat conservative result. This difference between calculated and measured results might be valuable as an additional margin of safety if it could be relied on in all cases, and it is important therefore to investigate some possible reasons for it.

It can of course be taken that the instruments used were capable of following accurately the fast pressure oscillations which were expected; no details are given in the text and it would be interesting if the author could indicate briefly the type of instrument used and its probable accuracy under the actual conditions.

The theoretical minimum pressure depends to some extent on the assumptions implicit in the surge diagram, Fig. 11. As noted in the text, the actual pipeline, which consists of a number of different pipes in series, has been replaced by the equivalent uniform pipeline determined by equs. (13) and (14). This may have contributed to the low value of the minimum pressure since in a somplete diagram the line A_0 $A_{3.5}$ (Fig. 11) would be replaced by a series of lines staggered either to the right or to the left. It would be interesting to know whether the author has attempted to draw a complete pressure surge diagram to include all or at least most of the changes in the pipeline, and whether that has given a value closer to the measured one.

Fig. 11 also indicates that the position of point $A_{3.5}$ and with it the minimum pump pressure are determined largely by the shape of the pump head-discharge characteristic α in this region. A comparison with Fig. 5 shows that this represents a very considerable extrapolation beyond the design point, well past the end of the pump characteristic given in Fig. 4. It would be interesting to know the method by which the author has carried out this extrapolation, and whether the curves in Fig. 7a obtained by Prof. R. T. Knapp were used in this connection too. The writer would also like to know whether in the author's opinion this part of the pump characteristic is likely to vary from one machine to another, particularly when the pump does not cavitate as in the present problem.

1. A.M.I.E.E., A.M.I.Mech.E., Mem. A.S.M.E. British Hydromechanics Research Association, Harlow, England.

Additional and highly important results appear to have been obtained by the author in another test at Tracy pumping plant when only one pump of a pair discharging in a common pipe line was stopped. To the writer's knowledge, no such experiments have been reported on an installation of this type, in which reverse flow through the pump occurs during slowing down before the butterfly valve has been closed. It would be valuable to have a comparison between calculated and measured pressures for this case also, if this has been investigated by the author, together with some indication of the method by which the distribution of flow between the running pump and the one slowing down has been determined.

DISCUSSION OF GRAPHICAL AND THEORETICAL ANALYSIS OF STEP-DRAWDOWN TEST OF ARTESIAN WELL PROCEEDINGS-SEPARATE NO. 362

RAPHAEL G. KAZMANN,[1] A.M. ASCE.—This paper is a lucid and thorough treatment of well-hydraulics from an eminently practical standpoint. It will be of immediate value to all ground-water engineers who have occasion to design wells, prepare specifications for alternate designs of wells, or compare well-fields with other methods of water collection such as infiltration galleries. Furthermore it spells out in considerable detail our need for research to supply deficiencies in design-data.

The total drawdown in a well is a major factor in determining the cost of bringing water to the surface of the ground. Well-life is a second factor of importance inasmuch as all calculations of annual cost are based upon this figure. The author has provided a basis of well-design which will tend to decrease drawdown and increase well-life and thus reduce the cost of water.

Clear indication is given that wells should be designed as laminar-flow devices. Apparently, for any given well, as a certain critical flow rate is exceeded, the drawdown is affected disproportionately by factors which the designer can modify: well-diameter and screen area, for example. The significant flow rate appears to be Q_c.

The author has evaluated Q_c for a screen 11-inches in diameter and 30 feet long. This value, 0.75 cfs, is of great practical importance. It enables computation of the velocity of water through the screen, or through the formation immediately outside the screen.

Although the total area of screen-opening is not given (and this would have been desirable), the porosity of material next to the developed well would probably range from 40 to 25 percent. Thus, the velocity of water just before it reached the screen surface (neglecting screen thickness) would range from about 0.022 ft/sec to 0.035 ft/sec, values significantly lower than figures generally cited in the literature.

In this connection it might be noted that an increase in the area of screen opening beyond 40 percent of the total screen area is not likely to produce a desirable decrease in velocity, all other factors remaining the same. Ideally the percentage of total screen area in openings should be the same as, or slightly greater than, the porosity of material immediately surrounding the screen. A highly uniform material might justify a larger percentage of openings than a less uniform material. Percentage of opening would normally range from about 15 percent to about 35 percent of the total screen area.

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It would be of value to the profession if the author could make available a table listing the following information for the several tests mentioned in the paper, and especially those listed on page 362-6: Q_c, B, C, P, T, S, r_n, length of screen, and percent of total screen area in openings. Such a table would show "order of magnitude" figures for the upper limit of velocity of laminar flow towards wells. An additional column in the proposed table might be labelled "v_c": the critical velocity below which laminar flow prevails from the aquifer to the inner surface of the well screen.

The magnitude of v_c will probably prove of importance in designing for increase of well-life. Experience has shown that, in incrustating waters, the more intensively a well is pumped, the shorter its life. When a ground water contains calcium bicarbonate it tends to deposit lime on the screen and in the interstices of the material immediately surrounding the screen. It has been observed that, as this process continues, the effective area through which water can flow is reduced, the drawdown increases, the rate of incrustation is increased disproportionately, and the well must be treated to restore yield or must be abandoned. Present theory indicates that where the cone of depression is steepest, that is, where the reduction in the internal pressure of the water is most rapid, the deposition of calcium carbonate occurs most rapidly. Equation of the reaction is approximately

$$Ca(HCO_3)_2 \dashrightarrow CaCO_3\downarrow + CO_2\uparrow + H_2O$$

It follows, therefore, that elimination of turbulent flow in the vicinity of a well should eliminate the abrupt pressure drop associated with turbulent flow and thus retard this chemical reaction. Well-life would thus be increased and maintenance costs greatly lowered.

Manifestly there are great opportunities for constructive work in the field of well-hydraulics under field conditions. Such studies should include a series of tests in different places under differing hydrogeologic circumstances to yield conclusive observations on factors of immediate practical importance such as Q_c and v_c. In conjunction with such studies it would be desirable to carry on studies of water quality to determine, if possible, in what manner the rate of $CaCO_3$ deposition (and deposition of other minerals) is related to the steepness of cone of depression. Such studies would result in immediate and tangible benefits to well-users and aid in the conservation of power and critical materials.

ABDEL-AZIZ I. KASHEF,[1] A.M. ASCE.—Mr. Rorabaugh raised in his paper a matter of major concern, namely, the indication "that the well radius may be more important in well design than has been considered in the past."

Under the steady-state condition the discharge varies as $\log \frac{r_e}{r_n}$ and this fact leads to the conclusion that the effect of well radius r_n is not

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important because the term $\log \frac{r_e}{r_n}$ varies little as r_n is changed. Under the non-equilibrium condition, the well known formula arrived at by both Theis[2] and Jacob[3] assumes that the pumped well has a theoretically infinitesimal diameter or a hypothetical line sink. In order to apply this non-equilibrium formula to an actual problem, Jacob suggested the determination of the drawdown curve for a line sink placed at the center line of the actual well, then the intersection of the well surface with the drawdown curve determines the accurate drawdown at this surface. This method was based on the fact that the rate of withdrawl of water from storage within the well casing is negligible.[4] This means that the effect of well radius is considered negligible under the non-equilibrium flow towards wells.

However, it had been shown by the writer[5] and others using numerical methods based on finite-difference equations that there is a certain effect of the well radius that leads to a variation of the drawdown—the discharge and other factors being constant—near the vicinity of the well casing compared to the Theis solution.

An example had been worked out by the numerical method and the Theis equation, and both solutions conformed more or less precisely only beyond 50 feet from the well center. It was concluded that, "This result was expected since the Theis formula is derived on the assumption that the well is represented by a hypothetical line sink. This assumption results in infinite values at the center line of the well. The Theis curve will not intersect the well surface at a point that corresponds exactly to the actual drawdown. In order to do this the well casing itself would have to be filled with the same kind of material as that through which the water percolates and not filled merely with water. The assumption of a line sink is not made in the numerical solution. The actual boundaries are considered."[5] The data chosen for this solved example were the same as that applied in the Grand Island Tests[6]: discharge = 540 gallons per min., transmissibility = 8.866 sq. ft. per min., coefficient of storage = 0.217, radius of well = 1.0 foot, the initial piezometric head was assumed to be 100 ft.

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2. Theis, C.V., "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of A Well Using Ground-Water Storage," Trans. Amer. Geophys. Union, Vol. 16, pp. 519 - 524, 1935.
3. Jacob, C.E., "On the Flow of Water in An Elastic Artesian Aquifer," Trans. Amer. Geophys. Union, Vol. 21, pp. 574 - 586, 1940.
4. Jacob, C.E., "Flow of Ground Water," Chapt. V in "Engineering Hydraulics," edited by H. Rouse, John Wiley, 1950.
5. Abdel-Aziz I. Kashef, Y.S. Touloukian and R.E. Fadum, "Numerical Solutions of Steady-state and Transient Flow Problems—Artesian and Water-table Wells," Engineering Experiment Station Bulletin No. 117, Purdue University, July 1952.
6. Wenzel, L.K., "Methods for Determining Permeability of Water-Bearing Materials," U.S. Geol. Surv. W.S. Paper 887, 1942.

The maximum difference between both solutions—Theis and the numerical solution—was at the well surface. The drawdown in feet at this surface and after different time intervals are shown in the given table. It is apparent that the Theis solution gives greater drawdowns as compared to the numerical solution. The difference ranges from about 80% of the drawdowns calculated by the numerical method in the early stages of pumping to about 50% in the late stages.

The writer's opinion agrees to the conclusions arrived at by Mr. Rorabaugh concerning the importance of the radius on well design. Moreover, there is no doubt that under the non-equilibrium conditions the difference in drawdowns aforementioned is due to neglecting of the well radius in the derivation of the Theis formula. More investigations are in urgent need in this respect so as to clarify the matter and arrive at general conclusions regarding the effect of well radii.

Drawdown (In Feet) at the Well Surface

Time Elapsed Since Start of Pumping (in Hours)	2	6	12	24	36	48
Drawdown Calculated by Theis Formula	6.030	6.742	7.191	7.640	7.903	8.090
Drawdown Calculated by the Numerical Method	3.325	4.243	4.758	5.256	5.543	5.745

CORRECTIONS TO FREQUENCY OF EXCESSIVE RAINFALLS IN FLORIDA PROCEEDINGS-SEPARATE NO. 366

Because additional data have been obtained for the rainfall at Jacksonville, Fla., Eq. 4e should be changed to

$$i = \frac{231}{31.2\,F^{-0.08} + t_a\,(F^{-0.58} + 0.46)}$$

This additional data will be included in the paper when it is printed in _Transactions._

DISCUSSION OF A NEW CONCEPT OF FLOW IN ROUGH CONDUITS PROCEEDINGS-SEPARATE NO. 390

V. L. STREETER,[1] M. ASCE.—In attacking the difficult problem of classifying both artificial and natural roughness in terms of the resistance function and wall Reynolds number the author is to be strongly commended. The lack of adequate experimental results does not as yet permit complete evaluation of the new concepts.

The author's analysis of quasi-smooth flow makes the tacit assumption that the kinetic energy per unit time flowing past a section through a vortex is the energy consumption of the vortex. There is probably a very slow mixing of the flow of the main stream with the fluid contained within the vortices, as the rapid shedding of vorticity would result in much larger energy losses in the pipe flow.

It is reasonable to assume that the rate of energy conversion to heat within a vortex in unit time depends upon the energy of the vortex. The expression for kinetic energy of a forced vortex ring is computed, then the vortex energy per unit weight of fluid flowing is developed. From this expression and the IIT experiments some conclusions as to energy dissipation within a ring vortex may be drawn.

The kinetic energy contained in a ring vortex of circumference πD and filament radius j/2, with angular velocity ω is

$$\int \frac{1}{2}\ v^2 dm = \int_0^{\frac{j}{2}} \frac{1}{2}\ 2\pi r dr\ \rho\pi D(\omega r)^2 = \pi^2 \rho D C_w{}^2 V_w{}^2\ \frac{j^2}{16}$$

The number of vortex rings per unit length of pipe is 1/2j for square groove roughness with land thickness equal to groove width. The weight flow through the pipe per unit time is

$$\rho g\ V \pi\ D^2/4.$$

The kinetic energy in the vortex rings per unit weight of fluid flowing per unit length of pipe is

$$\frac{\pi\ C_w{}^2 V_w{}^2}{8\ gDV}\ j$$

As V_w/V should not vary substantially, the energy in the vortices per unit length per unit time is proportional to $jV/(gD)$.

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Three sizes of square groove roughness were tested at IIT, with dimensions varying in the ratios 1:2:4. The test data, Fig. 7, indicates that the friction factor is practically the same for the three roughnesses, despite their large variation in relative roughness, and that they are constant for Reynolds numbers larger than 200,000. The energy in the vortices per unit weight of fluid flowing varies directly as the size j of the vortex and directly as the average flow velocity or wall velocity. Letting t be the fractional part of the energy converted into heat per unit time within the vortices,

$$\frac{k V j t}{gD} = \frac{f'}{D} \quad \frac{V^2}{2g}$$

where k is a proportionality factor. Solving for f' the resistance coefficient due to the vortices,

$$f' \sim \frac{j t}{V}$$

This shows that t must vary inversely as j and directly as V for constant f'. Hence the larger the vortex cross section (with correspondingly lower angular velocity) the less the rate of energy dissipation per unit of energy available in the vortex. Similarly the proportion of energy dissipated per unit of energy in the vortex increases directly as the wall velocity.

WALTER RAND,[1] A.M. ASCE.—The remarkable attempt, made in this paper, to analyze the flow over rough surfaces by taking into consideration the roughness characteristics h, λ, and s demonstrates at the same time the complexity of the problem and the difficulty of deriving equations that reflect all the aspects of the flow. The writer analyzed some of the equations related to the wake-interference flow in an attempt to find additional ways of interpretation and to reveal the significance of the numerical coefficients involved.

The logarithmic velocity distribution, adopted by the author as the basis of analysis (Eqs. 3 and 4), can be expressed in the simple form

$$\frac{v}{v*} = \frac{1}{k} \log_e \frac{y}{y'} \tag{26}$$

where y' is the value of y at which the logarithmic curve intersects the y-axis. In the hydraulics handbooks y' is given by

$$y' = \frac{\nu}{9.05\ v*} = \frac{r_o}{1.6\ R\sqrt{f}} \tag{27}$$

for the smooth pipe flow, and by

$$y' = \frac{h}{m} \tag{28}$$

for the rough pipe flow, with m = 30 for Nikuradse's sand roughness. Introduction of these values of y' into Eq. (1) will give the equations of von Karman for $\frac{v}{v*}$ and $\frac{1}{\sqrt{f}}$ (see Eqs. 4 and 5 in this paper).

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Analogous equations, involving the longitudinal spacing λ, can be derived for the rough flow by assuming that y' is proportional to λ :

$$\lambda = ny' \tag{29}$$

An alternative form of Eq. 7 would then be

$$\frac{v}{v^*} = \frac{1}{k}\log_e n + \frac{1}{k}\log_e \frac{y}{\lambda} \tag{30}$$

with

$$A = \frac{1}{k}\log_e n \tag{31}$$

The author's assumption that A is a universal constant would mean in this case that the ratio λ/y' should be a constant for all types of roughness (provided k is constant). However, as the roughness height h alone seems not to be sufficient to characterize the roughness fully, it is also difficult to believe that λ alone could be sufficient.

In fact, the author gives the values of A ranging from 6.8 to 9.3. The corresponding values of n will then range from 15.2 to 41.6. Such a scatter does not well warrant the assumption of a constant A. It seems that A is rather a function of all the roughness characteristics (h, λ, s, cross-sectional form of roughness elements) and could be constant for geometrically similar roughness. That means that the coefficient "1.75" in Eqs. 15 and 16 does not have a general validity and the value of resistance function for fully rough flow (high R_w) should rather be written in the general form

$$\frac{1}{\sqrt{f}} - \frac{1}{k\sqrt{8}}\log_e \frac{r_o}{\lambda} = \frac{1}{\sqrt{8}}\left(\frac{1}{k}\log_e n - \frac{3}{2k}\right) \tag{32}$$

for pipes, and $$\frac{1}{\sqrt{f}} - \frac{1}{k\sqrt{8}}\log_e \frac{r_o}{\lambda} = \frac{1}{\sqrt{8}}\left(\frac{1}{k}\log_e n - \frac{1}{k}\right) \tag{33}$$

for two-dimensional flow.

The author assumes that k is a constant (k = 0.4) for the total cross-section of the flow in the case of fully rough flow as c reaches zero at high values of the wall Reynold's number. In hydraulics the coefficient k is also accepted to be 0.4 in the case of smooth flow with the laminar sublayer covering the roughness elements. If k would be the same also for the transition zone between the smooth and fully rough flow, the conclusion would be that a variable n in Eqs. (32) and (33) could give the law for the resistance function in this zone. However, relying upon the results of velocity measurements (see Fig. 1) the author adopts a variable k (by $1/k = \Psi$) for the wall zone (Eq. 8), to derive the Eqs. 11 and 12. These equations reflect the correct trend of the resistance function for the transition zone in the case of uniform roughness. Nevertheless, even as the logarithmic velocity distribution curves (Fig. 1) show a tendency for a more pronounced slope near to the boundary, this tendency is artificially stressed by measuring y arbitrarily from a datum through the crests of the roughness elements. The choice of this datum would be inconsistent with the assumption of

the author that at high values of R_w the coefficient k is valid for the total velocity profile (c = 0). This assumption requires that the velocity should be zero at a finite distance y' over the crest of roughness that does not correspond to reality. In fact, various investigators (Nikuradse, Schlichting, Einstein, and others) have found that for fully rough flow a practically straight line velocity distribution can be plotted on logarithmic paper choosing the datum considerably below the crest of the roughness elements, rather somewhere near the centerline of these. Of course, the values of c and Ψ will depend on the choice of the datum from where y has been measured. Considering also the fact that the actual k can be different from 0.4 (in Fig. 1 the values of k can be found to be 0.33; 0.42; 0.35), the use of the Eqs. 11, 12, 15 and 16 seems rather difficult.

The author uses the Eqs. 15 and 16 to derive the limiting values of resistance function (3.52 and 2.64 on page 390-11). However, for the large value of c in $c\lambda = r_o$ the Eqs. 11 and 12 should be used. The value of resistance function for $\Psi = 0$ and $\frac{c\lambda}{r_o} = 1$ would then be $\frac{A}{\sqrt{8}}$ for both types of flow (pipe and two-dimensional). For a value A = 8.5 the value of the resistance function would be 3. However, the limit $\frac{A}{\sqrt{8}}$, with a value of A determined for fully rough flow can have no significance because a velocity distribution with $\Psi = 0$ over the total cross-section is not physically possible. The flow with the maximum value of the resistance function is very near to the smooth pipe flow, that will define the limit of the resistance function, and $\Psi = 1/k$ and c = 0 would be a more realistic assumption, even if it is not consistent with Eqs. 11 and 12. Moreover, the transition from smooth flow to fully rough flow depends possibly as much on the ratio $\frac{h}{\delta'}$ (where δ' is the thickness of the laminar sublayer) as on the ratio $\frac{\lambda}{\delta'}$ and a wall Reynold's number $\frac{R\sqrt{f}}{r_o/h}$ could be more relevant than $\frac{R\sqrt{f}}{r_o/\lambda}$ to determine the variation of the resistance function in the transition zone.

The writer hopes that these considerations are a farther illustration of the difficulties encountered in the study of flow over rough surfaces.

HARRY H. AMBROSE,[1] A.M. ASCE.—A review of data, published in the author's references for small commercial and artificially-roughened pipes and elsewhere (1) for large conduits, supplies convincing evidence to the fact that the effect of surface roughness characteristics upon the resistance to turbulent flow in conduits is an exceedingly complex phenomenon. The author is to be commended for his presentation of a concept of this phenomenon that helps to clarify many of its peculiarities—particularly that of the rising resistance coefficient

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curves associated with "wake-interference" flow. The characteristics of the three basic types of flow are well defined and appear to satisfy the trends of the available data.

The author has concluded that wake-interference flow, with its tendency toward constant resistance coefficients for moderate to high Reynolds numbers, is experienced infrequently in commercial pipes and that isolated-roughness or skimming flow is more likely to occur with attendant dependence of the resistance coefficient upon the viscous characteristics of flow for moderate, or even high Reynolds numbers. The writer anticipates a vigorous reaction to these conclusions but there is to be found much substantiating evidence. The appearance in the literature of data showing resistance curves closely paralleling the smooth-pipe curve has, in the past, been rather summarily dismissed and the surfaces yielding such resistance curves have been characterized as "wavy". This description appears hardly adequate in view of the similarity of results obtained from such widely varying surfaces as towing planes coated with ship-bottom paints (2), a large steel penstocks (3) and nominally smooth pipes with artificial depression-type roughnesses. It is apparent that more serious consideration must be given to isolated-roughness and skimming flows.

Although the writer finds phenomenological satisfaction in the three basic types of flow he disagrees with certain details of the relationships developed by the author. Every investigator of flow in rough pipes has been faced with the problem of deciding upon the correct diameter to be used in calculating the resistance coefficient and Reynolds number, and as a reference for velocity distributions. The choice is very important both in relating velocity distributions at wall distances within the magnitude of the roughness dimensions and in determining the resistance coefficient. In the latter usage, the value of the resistance coefficient varies directly as the fifth power of the diameter. The author has chosen to measure the diameter between the crests of the roughness elements at the opposite walls. This choice appears logical for closely-spaced elements. In the opinion of the writer, however, the correct diameter depends upon the type of roughness—particularly, upon whether flow in the space between the roughness elements can be considered as part of the mean flow. As an extreme, the author's rule would most certainly be incorrectly applied in large-scale isolated roughness consisting of protruding gaskets at flanged joints. The choice of diameter is also of particular significance in testing whether resistance effects of non-uniform elements are additive.

To be truly general, the resistance equations must allow for transition from smooth-pipe flow to each of the three basic types of rough-pipe flow and also for transition, whether abrupt or gradual, between the three basic rough-pipe flows. The author has not considered transition from smooth-pipe to isolated-roughness or skimming flow although this is at least as important as the wake-interference transition. Tests by the writer upon roughness elements consisting of cylindrical depressions in a surface that is otherwise smooth have yielded well-defined transitions from smooth-pipe flow to skimming flow. Three such transitions are indicated in Fig. 1. Noteworthy is the appearance of

the transition of curve B which gives every indication of the approach to constant resistance coefficient to often assumed for rough pipes. The marked downward trend of f for higher values of R, however, is very closely parallel to the smooth-pipe curve and thus reveals the transitional nature of the preceding portion of the curve. (The symbols used herein are identical with those defined by the author unless otherwise noted).

The writer is at a loss to understand the author's statement that for isolated-roughness and skimming flows the value of f is dependent upon R but not upon r_0. Perhaps independence from the parameter r_0/h is meant; however, the heretofore unpublished data of Fig. 1 show that r_0/h is a controlling factor in the transition range. Values for the roughness-element geometry for the three curves of Fig. 1 are given in Table I.

Table I

Geometry of Roughness Elements

Curve	r_0/λ	r_0/h	λ/h	d/h	a_d/A_s
A	7.75	31	4	1	0.047
B	7.75	31	4	2	0.189
C	3.75	15	4	2	0.181

(peripheral and longitudinal spacings are equal)

(The symbol d represents the diameter of the depression, a_d/A_s is the net ratio of depressed area to total surface area, and the other symbols are as defined by the author.)

The value of f cannot be independent of r_0 inasmuch as comparison of curves B and C shows the difference in the transition portions to be due to r_0/h (or r_0/y). Comparison of curves A and B indicates disagreement with the author's Eq. (23) since the additive f-value appears to be closely proportional to a_d/A_s and independent of λ/h. The claim may be made that the discrepancy is due to a variation in the product $c_w v_w$, but this would certainly appear fortuitous in view of the agreement of the above-mentioned trends with the results of tests on fourteen other surfaces (which, for lack of time and space, are not included).

For wake-interference flow, two distinct regions of velocity distribution are defined—the central or core region and the wall region. The author has assumed velocity distributions for each region in the development of the resistance equations. Certain discrepancies are to be noted. Inasmuch as Eq. (14) is independent of the Reynolds number of the mean flow it can be valid only for the range proposed by Nikuradse—that of "completely-turbulent"flow. Likewise, it is not obvious that A is a constant and independent of both R and R_w unless the resistance equation follows the quadratic law. In spite of these facts, the distribution of Eq. (14) is assumed to apply to the core region over the entire transition from smooth-pipe to rough-pipe flow.

If, as asserted by the author, the value of A is constant the core distribution of velocity will be governed only by the relative roughness spacing, whereas the wall distribution will be a function of both the type of roughness and the wall Reynolds number. For the assumed velocity distributions to apply over the entire transition range, therefore, the wall region must contain all of the viscous effects. Since for a smooth pipe the entire velocity distribution is dominated by the Reynolds number, the following confusing picture presents itself.

At the smooth-pipe end of the transition range the wall region encompasses the entire turbulent flow, the velocity distribution being governed by the wall Reynolds number and independent of the roughness characteristics. As the Reynolds number is increased there appears a core distribution at the center of the pipe governed solely by the relative roughness spacing although the wall velocity distribution enclosing the roughness elements is still primarily a function of the wall Reynolds number. Further increase of the Reynolds number causes the wall region to shrink and at the same time to be governed increasingly by the roughness characteristics rather than by the viscous effects. Finally, the wall region disappears and completely-turbulent flow ensues. In brief, if A is constant the effect of the roughness elements must begin at the pipe center and radiate toward the wall as the Reynolds number increases.

It does appear reasonable, however, that the velocity distribution should be divided into distinct zones which indicate the relative extent of influence upon the distribution of each governing quantity. As a matter of fact, Ross (4) has shown this to be necessary for the more simple case of turbulent flow over smooth surfaces. Ross divides the entire turbulent zone into an "inner turbulent" region which is said to be governed by a universal "law of the wall," an "outer turbulent" region governed by the spatial history of the turbulence, and a "blending" region which separates the inner and outer regions. Analogically, to the writer a more satisfying concept of the velocity distribution for transitional flow would be: (1) smooth-pipe velocity distribution as long as a laminar sub-layer persists; (2) with sufficient increase in the Reynolds number, generation by the roughness elements of eddies which initially would be weak and whose influence would be confined to the wall or inner turbulent region; and (3) with further increase in the Reynolds number, an increase in strength and extent of the eddies until, finally, the whole turbulent velocity distribution is dominated by the roughness elements. It is asserted that the influence of the surface roughness must extend from the boundary toward the center rather than the converse.

The foregoing criticisms do not lessen the writer's appreciation of the general aspects of the author's concept of turbulent flow in rough pipes—namely, that three basic types of flow are possible and that the spacing of the roughness elements is the controlling factor as to which type of flow will attain in a given case. It is believed that the profession would do well to reanalyze existing data in the light of this concept and to use it as a guide in future research on the subject.

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3) Burke, Maxwell F., "High Velocity Tests in a Penstock," Proceedings Separate No. 297, ASCE, Vol. 79, Oct. 1953.

4) Ross, Donald, "A Study of Incompressible Turbulent Boundary Layers" A thesis submitted to Harvard University in partial fulfillment of the requirements of the Ph.D. degree, June 1953.

HENRY M. MORRIS,[1] A.M. ASCE.—It has been somewhat surprising to the writer that, in view of the rather sharp departure of the new concepts from previous methods in a field of wide interest in hydraulics, there has been such relatively little criticism of the paper. It is hoped that this fact indicates the new concept will prove generally acceptable and useful to the profession. The comments and suggestions of the discussions by Messrs. Streeter, Rand, and Ambrose, together with the generally favorable attitude taken toward the paper, are greatly appreciated by the writer.

Dr. Streeter suggests an alternate analysis of quasi-smooth flow friction factors, which indicates that the proportion of groove vortex energy actually being converted into heat varies inversely as the groove width and directly as the flow velocity, assuming constant friction factor.

However, it appears to the writer that there is a dimensional inconsistency in this analysis. Dr. Streeter derives an equation for the total kinetic energy, in ft.-lbs., contained in the groove vortex, and then divides the result by the weight-rate of flow, in lbs. per sec. The resultant expression, which Dr. Streeter calls the kinetic energy in the vortex rings per unit weight of fluid flowing per unit length of pipe, is thus actually the vortex energy. for each lb. per second flowing, for a unit length of pipe. The dimensions for his expression are therefore (ft.-secs.) per ft., and the quantity seems to have no pertinent physical significance as it stands. Also, it seems questionable to attribute the entire flow friction factor, as measured in the I.I.T. tests, to the energy of the vortex. A substantial part of the energy consumption of the flow is due to the normal "smooth-pipe" action, whereby vorticity is generated and shed into the main flow at the pseudo-wall consisting of the land areas of the threads and the upper limbs of the groove vortices. It is only the additional energy loss, above the normal smooth-pipe loss, that is attributable to the groove vortices.

It seems reasonable to assume, as the writer did in the derivation of Eq. (22), that the entire vortex energy per unit time is being continuously converted into heat, under equilibrium conditions. However, the vortex itself is not dissipated, as it continually receives an equivalent supply of power through attrition of the bulk flow with its upper limb.

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1. Prof. and Head, Dept. of Civ. Eng., Southwestern Louisiana Inst., Lafayette, La.

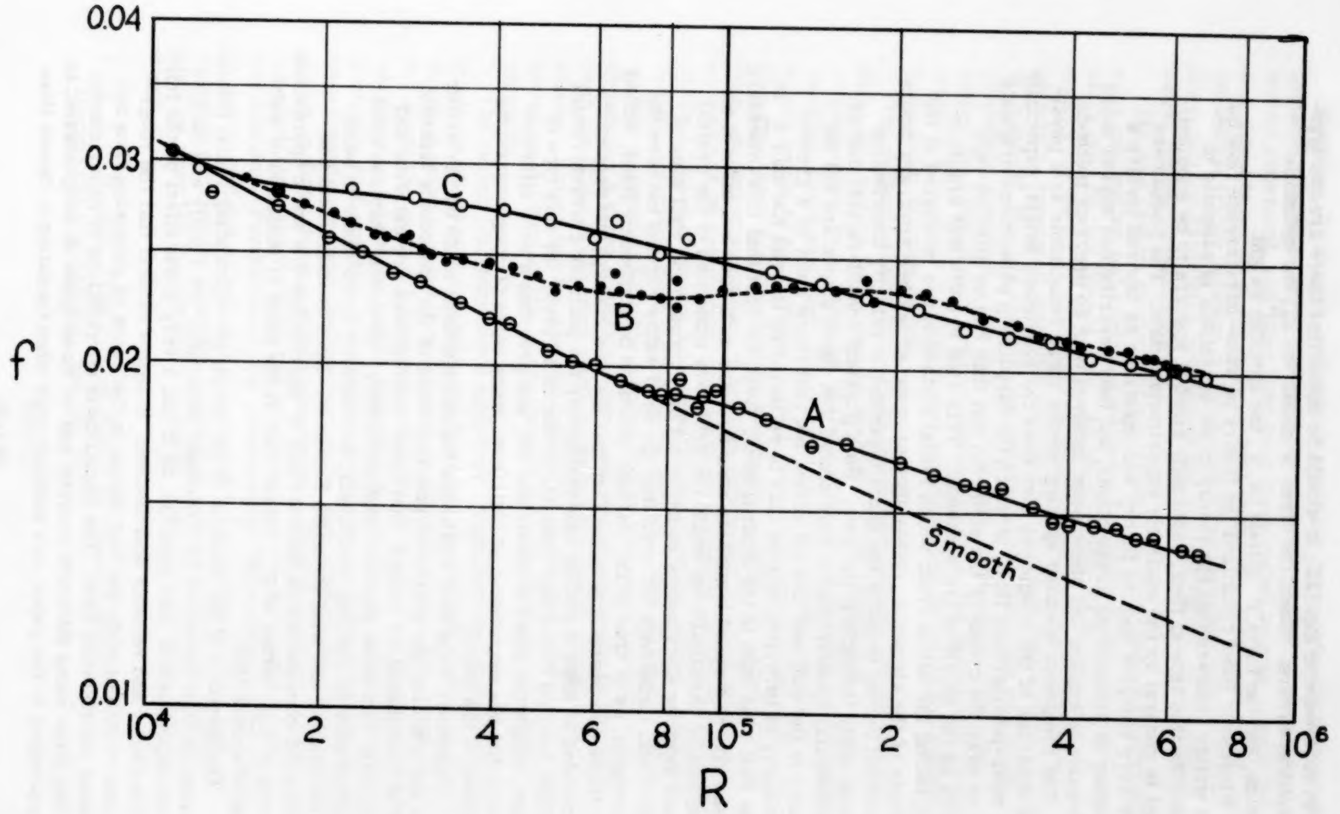

Fig. 1

In reference to Eq. (22), it should be noted that there are two typographical errors. Thus, the term R should be R_h, the hydraulic radius, and the term ν should be γ, the specific weight.

Professor Rand criticized the theory of wake-interference flow but the writer believes that the theory in its essentials is adequate to describe this type of flow, especially since it appears to be substantiated in general by the available experimental data. The phenomena are very complex and the theory and equations as derived involve a number of simplifying assumptions, but they nevertheless appear more adequate to explain the phenomena involved than do previous methods.

The roughness element spacing seems more reasonable as a physical measure of the roughness than does the roughness height, especially in wake-interference flow, in which the turbulence phenomena originate at or near the crests of the elements and thus can be affected very little by the height of the elements. This also is sufficient argument for taking the datum from which radial distances are measured at the crests of the elements, rather than at some altogether arbitrary datum selected purely to make the same logarithmic velocity distribution slope apply throughout the pipe. Both physical considerations and experimental measurements have shown that these slopes are not the same in the wall and central zones, but that there must be a greater intensity of turbulent mixing than is reflected by the von Karman $\underline{k}$ in the wall zone than in the central zone. Even the fact that this procedure must ignore a small transition zone near the wall does not make it as physically unrealistic as would the arbitrary extension of the central zone velocity distribution equation not only through the wall zone of abnormal turbulence but even into the dead water regions between the elements, as is true of the methods defended by Professor Rand. Added to these considerations is the fact that the new concept and procedure provides at least a partial explanation for the peculiar characteristics of the friction factor and resistance function curves for this type of flow, which the older methods did not, and provides a very effective basis for the correlation of data from many types of roughnesses, as shown in the paper.

Professor Rand also criticizes the assumptions made by the writer in determining the maximum possible value of the resistance function. It is recognized, of course, that these assumptions are extreme and actually represent physical impossibilities. However, they are quite reasonable as limiting conditions, and actually come close to being realized for low wall Reynolds Numbers. Both measurement and physical considerations indicate that Ψ approaches 1/k and c approaches zero at high values of R_w, rather than at the point of initiation of wake-interference flow.

The question of the location of the datum for calculating radial dimensions is also discussed by Professor Ambrose. He rightly emphasizes the importance of this question, as it has a very great effect on the friction factor and velocity distribution. It seems evident that the roughness crests provide the best datum in the cases of skimming flow and wake interference flow. The remarkable correlations of experimental data from many different sources and on many types of roughnesses, as discussed in the paper, are possible only when the datum is chosen thus.

However, Professor Ambrose is probably correct in pointing out that isolated roughness flow, particularly when the roughness elements are widely separated, and when they have large radial heights, should be described in terms of the wall-to-wall diameter. The case of the transition region between isolated roughness and wake-interference flow, and the associated problem of the correct datum, is one which needs further investigation.

The new data on skimming flow obtained by Professor Ambrose are very interesting and provide a welcome extension to the I.I.T. and other data on this type of flow. An item of interest is that the f-R curves for the I.I.T. roughnesses, which were strip-type roughness elements, separated from the smooth-pipe curve at much lower values of Reynolds Number than did the curves for the Ambrose spot-type roughness elements. The inference is that, in skimming flow, the larger the ratio of wall area to depression area, the larger will be the value of R at which the depression vortex begins to contribute substantially to the friction factor.

The writer's statement that, in isolated-roughness and skimming flow, the friction factor was independent of the radius was not, of course, meant to be taken as excluding the effect through the Reynolds Number, which was explicitly stated to be functionally related to f. It was meant to say that the friction factor was independent of the relative roughness and the relative roughness spacing for these types of flow. Apparent variations of friction factor with relative roughness as obtained on commercial pipe tests can probably be attributed to incorrect choice of radius for computation of friction factor. This had, in fact, previously been suggested by Piggott,[1] who, in his extensive studies of commercial pipe friction factor data, noted that the f-R curves for a given roughness type could usually be made to coincide, even for pipes of different radii, provided the proper datum was chosen. Except for the unusual conditions mentioned previously, this would most likely be the datum through the crests of the roughness elements.

The cylindrical depression spot-type roughness elements tested by Professor Ambrose are different from any coming previously to the writer's attention, and it is difficult to classify them. His curve A, for which the ratio d/h (corresponding to the writer's j/h) was 1, seems certainly to follow the trends indicated in Eq. (23) for skimming flow. However, this equation was derived primarily for a groove-type depression extending around the pipe periphery, and reflects the energy expenditure in maintaining a stable ring vortex. This is a somewhat different situation from the vortices that would be formed in cylindrical spot depressions. Furthermore, the roughnesses represented in his curves B and C had d/h values of 2 and, as indicated by the writer, it is doubtful whether the depression vortices are maintained as stable entities when the groove width becomes materially greater than its depth.

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1. R. J. S. Piggott, "The Flow of Fluids in Closed Conduits," Mechanical Engineering, Vol. 55, 1933.

Consequently it is doubtful that skimming flow was produced at all on roughnesses B and C. The typical form drag phenomenon of a continued generation of vortices in the wake would certainly extract more flow energy than the stable vortices of skimming flow, and this would most probably account for the increase in friction factor as between roughnesses A and B, rather than the greater proportion of depression area to wall area, as suggested by Professor Ambrose.

It is therefore likely that, whereas roughness A produced skimming flow, roughnesses B and C produced either wake-interference or isolated roughness flow, with wakes being produced at both extremities of each depression. In view of the peculiar character exhibited by Curve B, there is the possibility that roughness B tended to produce wake-interference flow at low Reynolds numbers, changing to isolated roughness flow at higher Reynolds numbers, with the result that curves B and C essentially coincide thereafter as would be indicated by Eq. (19) for isolated roughness flow.

Professor Ambrose mentioned other experimental data which he had obtained. It is hoped that, when these are published, they will be able to clear up these problems.

The writer believes Professor Ambrose, in his remarks about the velocity distribution equations in wake-interference flow, may not have grasped the full significance of the wall region of abnormal turbulence. It is not strictly correct to speak of this sort of flow as transitional between smooth-pipe and rough-pipe flow, or of the latter as being characterized by "complete turbulence," because such terminology tends to imply that this transitional flow is not itself fully turbulent, but partly controlled by boundary layer effects remnant from smooth-pipe action. Actually, the energy losses in this type of flow are proportional to a power of the velocity greater than 2, as shown by the rising f-R curve, and thus the turbulence in the wall region should be described as abnormal turbulence, as distinct from the normal turbulence characterized by the quadratic resistance law, and also by the von Karman constant k. There is, of course, a short transition from smooth-pipe phenomena to these phenomena of abnormal turbulence; this transition is represented by the portion of the "resistance function—wall Reynolds number" curve which gradually diverges from the smooth-pipe curve, finally becoming horizontal. The long rise in the curve which then follows, finally becoming horizontal again, is the zone of abnormal wall turbulence.

The wake-interference flow theory and equations, as developed in the paper, deal of course only with the part of these phenomena represented by the rising and ultimately horizontal f-R curve. Isolated roughness flow would occur at lower Reynolds Numbers, represented by the portion of the transition curve from the point of departure from the smooth-pipe curve to the low point in the dip; when wake-interference flow is initiated, the f-R curve begins to rise. This reflects the fact that the wall zone has been transformed from one controlled primarily by phenomena in the laminar boundary layer, which by then has vanished, to one of abnormal turbulence caused by the "wake-interference" phenomenon. The velocity distribution near the wall in smooth-pipe

flow is characterized by a slope corresponding to a number greater than $1/k$; the wall velocity distribution in wake-interference flow has a slope ψ which is less than $1/k$, caused by the abnormal intensity of turbulence in that region.

It has been found and generally accepted that the dimensionless velocity distribution equations in the central region of the pipe have the same slope (corresponding to the von Karman k) regardless of Reynolds Number and for both smooth and rough walls. This is the zone denoted by Ross (as cited by Professor Ambrose) to be the outer turbulent zone, governed by the spatial history of the turbulence. In this zone, the vorticity generated at the wall or elsewhere, by whatever means, whether at a laminar boundary film or by wake vorticity, has been broken down by mutual attrition and momentum transfer to "normal" turbulence. It is surely reasonable that the central zone turbulence would also be "normal" during the transition throughout the full range of wake-interference flow.

As soon as the transition from smooth flow to the quadratic law flow is complete (indicated by the low point of the dip on the f-R curves), then true wake-interference flow begins. From this point on, the wall zone turbulence is governed by the roughness elements entirely (more precisely by the element spacing and the form of the wakes behind them, which latter is a function of the geometric form of the elements throughout the transition). It seems reasonable that, for this regime, the term A should be a constant as well as k. Reasons for this were discussed in the paper, as well as the experimental evidence that it was so.

Thus, it appears to the writer that most of Professor Ambrose's discussion of the phenomena of transition from smooth to rough pipe flow are quite correctly applicable to the first stage in the transition, but not to the second stage. At the initiation of this stage, the influence of the wall roughness has already extended completely to the center of the pipe, and the boundary constant A has become established as a characteristic of the core velocity distribution, although only in the central pipe zone has the wall turbulence been reduced to the normal turbulence characterized by k. Then, as the Reynolds Number increases further, the central zone extends toward the wall, both by shifting of the boundary of the two zones toward the wall and also by more rapid reduction of the abnormal wall turbulence to normal turbulence even in the wall zone, as evidenced by the approach of the velocity distribution slope in the wall zone to $1/k$, the slope of the core velocity distribution.

There is no question but that the phenomena of flow in rough pipes are many and complex. There is much room for additional investigation, especially in the various boundary zones. The present writer hopes that the new concepts presented in this paper will help to clarify at least some of these phenomena and point the way to further research, and at the same time will prove useful in practical engineering design procedures. Again he wishes to thank the writers of the discussions for their interest and very material contributions to that end.

DISCUSSION OF BACKWATER EFFECTS OF OPEN CHANNEL CONSTRICTIONS PROCEEDINGS-SEPARATE NO. 413

PAUL V. HODGES,[1] M. ASCE.—A very useful and simple method for the determination of backwater effects caused by different types of channel contractions is presented in this paper. The base curves and coefficients are defined in the authors' reference (2). When the drop in water-surface, Δh, for a particular contraction has been determined by field observation, the actual backwater, h_1^*, caused by the contraction, can then be determined from base curves similar to the authors' Figure 9, which would give the backwater ratio $h_1^*/\Delta h$ for different degrees of contraction.

According to the tests made in the investigation, it is shown that the variation of L/b has no effect on the backwater ratio. In authors' reference (2) the coefficient C is shown to vary with L/b, and in the authors' Figure 12 the backwater ratio is shown to vary with coefficient C. Consequently it appears that the backwater ratio should also vary with L/b.

The base condition is defined as a vertical-faced constriction with square-edged abutments, and the curves in authors' Figure 9 evidently refer to this condition only. It is inferred that the relation between $h_1^*/\Delta h$ and m would be different for each type of constriction.

By assuming different values of Δh and y_3 for a particular type and size of bridge opening, the discharge Q can be computed by the equation in authors' reference (2), and thus the relation between backwater h_1^* and the discharge can be determined to fit the assumed conditions. Under actual conditions of flow, however, the relative values of Δh and and y_3 would be governed by channel characteristics, both upstream and downstream from the contraction, and consequently the discharge could be computed only after the actual determination of Δh and y_3.

This method, as presented by the authors, could often be used to compute the backwater from channel obstructions caused by bridge piers. Backwater formulas, based on many tests, have been derived for the computation of backwater effects of bridge piers, of which the Nagler formula appears to be the most satisfactory. See writer's references (5) and (6). Using the authors' notations, Nagler's formula is

$$Q = 8.02\, K_n b[y_4 - \theta V_4^2/2g]\, [h_1^* + \beta\, V_1^2/2g]^{\frac{1}{2}}$$

The values of coefficients θ and β are given in references (5) and (6).

The relations between drop in water surface Δh, backwater h_1^*, degree of contraction, m, and discharge as computed for a certain type

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and size of bridge opening, are given in the following table. This table shows the computed value of K_n, which when used in Nagler's formula will give the backwater h_1^*.

Writer's Table 1.—Relation Between Contraction, Backwater and Discharge

m	Δh	$h_1^*/\Delta h$	h_1^*	K_n	Q
20	1	.34	.34	.886	12,900
20	2	.34	.68	.916	19,100
30	1	.45	.45	.861	10,900
30	2	.45	.90	.884	15,900
40	1	.54	.54	.840	9,470
40	2	.54	1.08	.860	13,800
50	1	.62	.62	.810	8,510
50	2	.62	1.24	.822	12,400

Type.—Vertical face constriction with square-edged abutments.

b = 90 feet n = .025 $y_1 = y_3 + \Delta h$

L = 30 feet $y_4 = y_3 + \Delta h - h_1^*$

y_3 = 15 feet L/b = .333

A_3 = 1350 sq. feet y_3/b = .167

Q = Discharge as computed by the equation and coefficients given in authors' reference (2).

The authors indicate that the backwater ratio is not influenced by a variation in the paramters y_3/b and L/b, but only by the degree of contraction m. Under these conditions the method presented in this paper would have wide and easy application compared to earlier methods which are much more complicated and involved with discharge formulas.

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CARL F. IZZARD,[1] A.M. ASCE.—This paper,together with the companion paper by Kindsvater and Carter (1), reports the first comprehensive effort to obtain model experimental data from which to develop a

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method for computing backwater caused by a channel constriction such as a bridge. The contribution reflects the judgment of the U. S. Geological Survey that small-scale hydraulic model tests offer a sound means of exploring the significance of the many variables involved in the problem of indirect measurement of discharge by the contracted opening method. The research has been intelligently organized and aggressively conducted. The reports published to date (including this paper) have been confined to conclusions drawn from the experimental data. While not stated in the paper, the Survey is making field measurements of backwater and discharge as rapidly as possible in order to verify the results of the model tests.

One of the outstandihg values of the work accomplished is the knowledge of where to take significant measurements of contracted flow. The verification of the laboratory results by prototype measurements will be followed with great interest not only by engineers who want to compute discharge for a known backwater but also by engineers who must estimate backwater for a proposed bridge or other constriction of an open channel.

This discussion will be limited primarily to the problem from the viewpoint of the highway engineer since this is the field in which the writer is engaged.

Viewpoint of the Highway Engineer

The primary objective of the highway engineer is to build a structure which will safely handle the vehicular traffic at a minimum annual cost. That cost should include not only capital and maintenance cost but also costs for less tangible items such as flood risk (defined as flood damage cost multiplied by probability of occurrence). The flood damage cost is directly related to the amount of backwater both with respect to damage to abutting property, which sometimes entails litigation, and to destruction of parts of the highway itself. The frequency of a flood which overtaxes the highway structure determines the period of time over which the probable damage must be charged off as flood risk cost. Depending on the nature of traffic and average daily count the flood risk cost may also include substantial charges for interruption to, or interference with, movement of traffic. Where approach roadways are designed for overflow, backwater and duration of flood as well as frequency are important.

One important distinction must be noted between the objectives of the hydrologic engineer and the highway designer. The former is expected to achieve a fairly high standard of accuracy in his estimate of the flood discharge as computed from backwater and that answer is his end result. The highway designer, on the other hand, puts the computation in reverse and wants to know how much backwater can be expected for floods of various frequencies whose magnitudes can probably be estimated not closer than twenty percent plus or minus (unless a gaging station with long record happens to exist somewhere nearby on the stream). Obviously then the highway engineer does not need to work to the close tolerances expected of the engineer who is gaging streams.

This distinction should be borne in mind to avoid creating the impression that the writer is unduly simplifying a complex problem, or casting reflections on the U.S. Geological Survey for being too meticulous. At this stage of development the research must be as thorough as possible.

The following analysis was possible only because of the cooperation of the authors who made available all the basic data recorded from the tests conducted at the A. N. Talbot Laboratory of the University of Illinois.

Analysis of Data for Base Condition

A constriction placed in an open channel requires that the velocity in the constricted section be greater than in the normal section. The forces producing this change in momentum neglecting forces to overcome normal resistance are the unbalanced forces represented by the total pressure at some upstream section less the total pressure against the embankment constricting the flow. Since the water elevations against the embankment are not known with any precision and are decreasing where flow is accelerating, exact mathematical analysis in this reach is out of the question, although it does work fairly well in the decelerating reach below the constriction as demonstrated in the paper.

This concept does lead to an hypothesis that the increased depth in the approach section relative to the normal depth of flow should correlate with the change in momentum. The change in momentum obviously is dependent on the conveyance of the constricted section relative to the conveyance of the approach section, which is essentially the same as (1 - m). A convenient form of plotting the experimental data as a means of testing this hypothesis is shown in Fig. 13 in which the ordinate is the depth ratio first mentioned.

The abscissa in Fig. 13 is the velocity head in the constricted section relative to the "normal"depth. In this case, since the laboratory flume was level, "normal" depth means the depth y_n at the location of the constriction with the constrictions removed. The velocity, of course, is the discharge divided by the area at this "normal" depth. The difference in y_n at Sections 1 and 3 can be considered negligible in these tests.

The experimental data for one size of vertical-face constriction with square-edged abutment (base condition) define three curves for three values of roughness. The maximum difference between extreme values of n is on the order of 2.5 percent of the normal depth, and is therefore not important from the highway viewpoint. Also the length of constriction L measured along the flow has little effect so long as L is less than one-third of the width of opening b. The curves also apply reasonably well to the non-rectangular section (Fig. 5) for a comparable value of m as shown by the two points so designated in Fig. 13.

For other values of b the change in momentum for a given "normal" flow would be different. Figure 14 shows curves for values of m of 0.21, 0.41, 0.61 and 0.81; no ordinate scale is indicated since the object is to show the relative position of the curves and not to provide a working chart. The curve for m = 0.41 corresponds to an intermediate roughness value in Fig. 13.

Application of Analysis

The apparent correlations obtained from analysis of the data from the scale model are encouraging but not conclusive. The indications are that a graph such as Fig. 14 might be used as a simple means of estimating relative backwater caused by bridges of different length. A separate set of curves might be developed for each of the commonly-used bridge types. Adjustment factors for skew, eccentricity, and piers might be computed.

Comparisons of backwater computed by this short-cut method and by the procedure recommended by the authors for certain hypothetical cases have given reasonable agreement. The agreement is better if the depth ratio at the upstream point is considered to apply to the mean depth at that section rather than the depth above the stream bed.

The validity of any computational method based on the model tests cannot be firmly established until prototype measurements of backwater and discharge are available for verifications. It may be noted that actual stream crossings will rarely involve depths of flow as deep relative to the flow width as used in the model test.

While the use of a level flume simplified testing at both the laboratories, the effect of stream slope on backwater is obscured. It seems reasonable to assume that the increment in head loss over the normal resistance loss in the approach reach with no constriction is included in the empirical evaluation of the ratio y_1/y_n. This is a weak point in the writer's analysis since the model is obviously not geometrically similar to any usual prototype. The separation of losses as provided by the authors may be more nearly correct for some situations.

The Backwater Ratio

The backwater ratio as defined by the authors is apparently independent of the Froude number. The ratio y_1/y_n in Fig. 13 is a function of F for the constricted section at "normal" depth. This follows because the abscissa is

$$V_n^2 / (2g\bar{y}_n) = F^2/2.$$

Since $y_n + h_1^* = (y_1/y_n)\, y_n$ by definition, it also is a function of F. Further if the last two terms in Equation (1) are of the same order of magnitude they tend to cancel out and thus Δh is also a simple function of the Froude number. This may explain the lack of variation in the backwater ratio with F, but it also raises a question.

That question is —if $h_{f_{1-3}}$ is large compared to the approach velocity head term in Eq. (1) (and it is not so in the model tests) will the backwater ratio still apply?

Figures 11 and 12 can be readily combined for a given value of n into a single graph in which the backwater ratio is plotted against the contraction ratio for equal values of the variable C_c. The experimental

data, however, is quite meager for bridge types other than the base condition as is indicated by the scatter of data in Fig. 12.

It is hoped that the authors will be perfectly frank in disclosing any serious defects in the form of presentation of data illustrated in Fig. 14. Such a graph gives a direct answer for backwater knowing the stage-discharge relation at the site, the waterway area under the bridge for any stage, and the conveyance of the bridge section in relation to the conveyance of the approach section. It does not give a measure of the depth y_3 which is the depth in the eddy area at the downstream side of the abutment and usually less than the normal depth of the unconstricted stream. But that depth, although very important for indirect discharge measurements, appears to have no practical significance on bridge design. The model tests indicate that water surface elevations at other points along Section 3 will usually be slightly higher than elevation h_3.

Any one experienced in measuring velocities and depths of flow at bridges will know that the bed as a whole (not considering localized scour around piers) may lower during high stages of flow if the stream bed is alluvial and especially if it is non-cohesive. This increase in area apparently would reduce the required velocity but what would be the effect on backwater? This is one of the many problems which still need to be investigated, and may justify use of admittedly approximate solutions of the backwater depth at the present juncture.

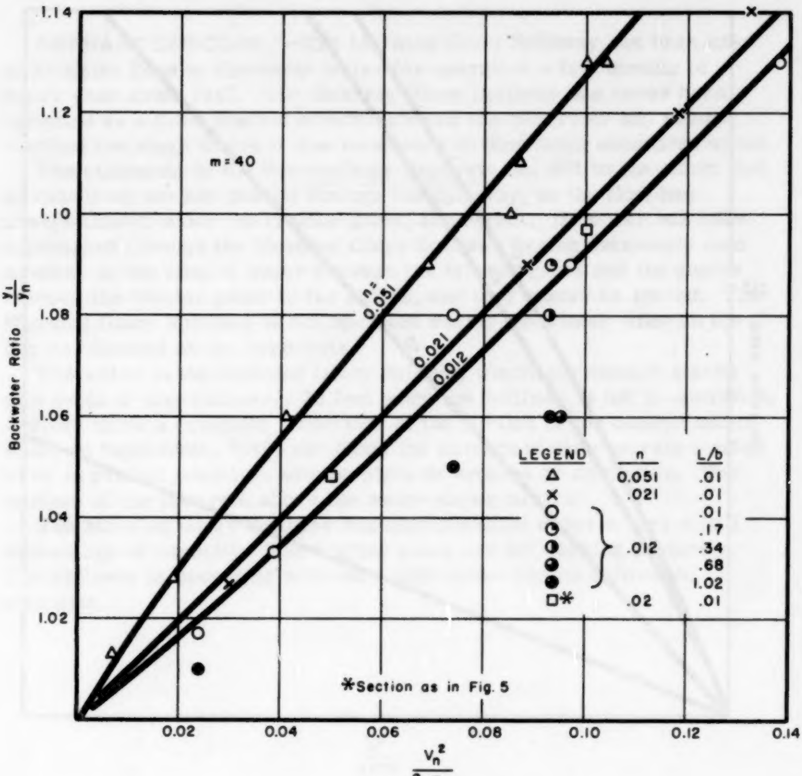

FIG. 13 - BACKWATER DEPTH RELATIVE TO NORMAL DEPTH, VERTICAL FACE CONSTRICTION WITH SQUARE-EDGED ABUTMENTS, b = 2.95 FEET

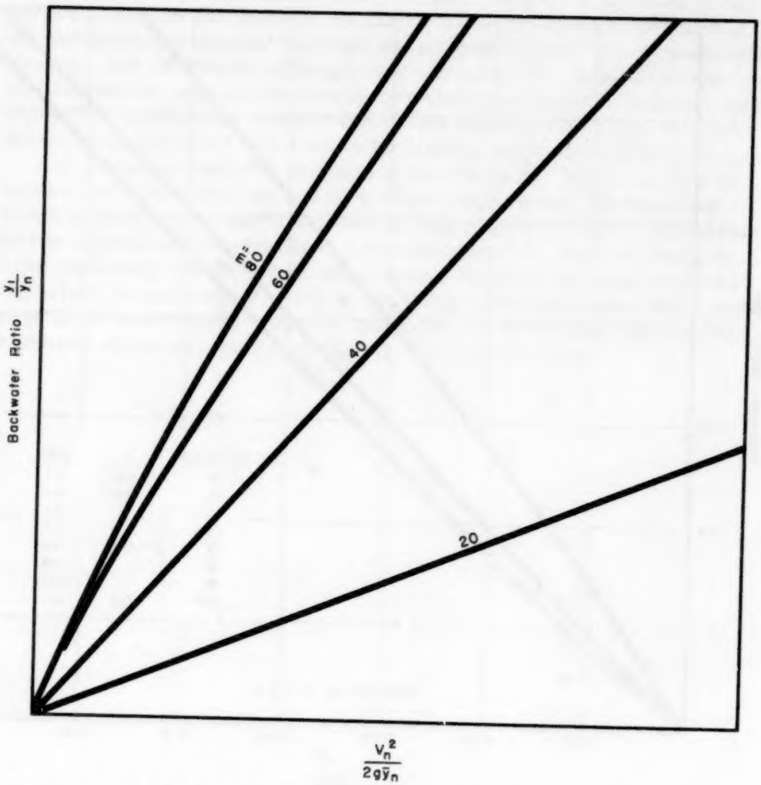

FIG. 14 - BACKWATER DEPTH RELATIVE TO NORMAL DEPTH, VERTICAL FACE CONSTRICTION WITH SQUARE-EDGED ABUTMENTS, b = VARIABLE

DISCUSSION OF MORNING-GLORY SHAFT SPILLWAYS: PROTOTYPE BEHAVIOR PROCEEDINGS-SEPARATE NO. 431

BERNARD DONCLON,[1]—The Morning Glory Spillway has been used at Kingsley Dam to discharge water for operation a few months of every year since 1947. The Morning Glory Spillway has never been operated as a flood control structure since the reservoir has never reached the stage where it was necessary to discharge excessive water.

The statement in the Proceedings-Separate No. 431 to the effect that no debris or ice has passed through the Spillway, as the flow has always flowed under the tractor gates, is correct. No water has been discharged through the Morning Glory Spillway during extremely cold weather as the film of water between the tractor gates and the guides freezes the tractor gates to the guides, and they cannot be moved. The Morning Glory Spillway is not operated during the winter when an ice cap has formed on the reservoir.

The water in the Morning Glory Spillway discharge conduit stands at a depth of approximately 20 feet when the Spillway is not in operation. A diver made a complete inspection of the portion of the conduit under water in September, 1949, and found the surface of the concrete conduit to be in prefect condition with no signs of erosion or cavitation. The surface of the concrete above the water shows no wear.

The Morning Glory Spillway has only operated under a very small percentage of capacity. The tractor gates are all working perfectly. The Spillway is operating with very little noise and no noticeable vibration.

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DISCUSSION OF MORNING-GLORY SHAFT SPILLWAYS: DETERMINATION OF PRESSURE-CONTROLLED PROFILES PROCEEDINGS-SEPARATE NO. 432

MAXWELL W. WHITE,[1] MURRAY B. McPHERSON,[2] J.M. ASCE.—The characteristics of shaft or morning-glory spillways in important structures are determined either by a model test of the actual structure involved or by an application of principles arrived at by previous studies on model shaft spillways or by more fundamental studies on sharp-crested circular weirs. The scaling up of results from these model tests to prototype conditions requires that the "scale effects" (i.e. factors affecting flow over model weir which are not present in the prototype) either be negligible or else be allowed for in the calculation.

The results given for discharge coefficients in Mr. Wagner's paper cover spillways designed for maximum design heads from H_s/D (where $D = 2R$) greater than 0.10 up past the submerged range. However, not many spillways have been designed to perform submerged. Of the 18 morning-glory spillways for which details have been published[3] only 4 are designed to perform submerged, 6 for ratios of H_s/D between 0.10 and 0.20 and 8 for ratios of H_s/D less than 0.10.

Tests were carried out on sharp-edged circular weirs in the Hydraulic Laboratory, Fritz Engineering Laboratory, Lehigh University between 1949 and 1954.[4] The project was sponsored by the Department of Civil Engineering and Mechanics. Professor W.J. Eney is Director of Fritz Laboratory, and Head of the Department. The tests were aimed at determining

(1) the head-discharge relationship (i.e. between basic parameters Q, H and D);
(2) the effect on this relationship of fluid properties such as surface tension and viscosity, particularly for small heads;
(3) the point at which weir flow changes to flow through a reentrant tube.

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2. Associate Professor of Civil Engineering, Lehigh University, Bethlehem, Penna.
3. "Morning-Glory Shaft Spillways: Prototype Behaviour," by J. N. Bradley, Proceedings A.S.C.E. Separate No. 431, April, 1954.
4. "Characteristics of Sharp-Crested Circular Weirs," by M. W. White, G.M. Brey and R.G. Dittig. Report No. A-1342, Fritz Engineering Laboratory, Lehigh University, April 1954 (Available on loan).

These results were then compared with those of other investigators to extend the range and value of the results. A brief survey of the investigations and of the results obtained, is reported here.

The present results were not extended to pressure-controlled undernappes mentioned in Mr. Wagner's paper, all tests being conducted with the undernappe aerated at atmospheric pressure and with the nappe springing clear from the crest. The approach conditions were also fixed at uniform radial flow.

From preliminary tests, it was found, in agreement with the paper, that, as long as P was greater than D, the effects of the velocity of approach head and the ratio H_s/P were negligible on the discharge coefficient. Accordingly, this factor was eliminated from subsequent testing, in all cases P being greater than D.

With these variables eliminated, the remaining factors can be grouped into 4 significant dimensionless parameters.

(1) $$\frac{Q}{H_s^{1.50}\, D^{1.00}\, g^{0.50}} = \frac{C\pi}{g^{0.50}}$$

(2) $$\frac{H_s}{D}$$

(3) $$\frac{H_s^{1.50}\, g^{0.50}}{\nu} = N_R, \text{ a Reynolds No.}$$

(4) $$\frac{\gamma H_s^{2.00}}{\sigma} = N_W, \text{ a Weber No.}$$

and $$\frac{Q}{H_s^{1.50}\, D^{1.00}\, g^{0.50}} = f\left|\frac{H_s}{D}, \frac{H_s^{1.50}\, g^{0.50}}{\nu}, \frac{\gamma H_s^{2.00}}{\sigma}\right|$$

The first parameter is a theoretical coefficient of discharge. Dimensional analysis shows that this coefficient is a function of H_s/D, which expresses the contractive effect, a Reynolds No. which expresses the effects of viscosity and a Weber No. which expresses the effects of surface tension.

In the tests at Lehigh University only one fluid (water) was used, therefore the last two parameters, N_R and N_W, could not be varied independently, since both are a function of fluid properties and head, H_s, only. The effects of the two factors, then, could not be presented generally for any fluid nor could their individual effects be separated. For this reason the results presented here apply only to water, and, in the investigations, the two parameters, N_W and N_R, can be represented as a function of H_s only, for water.

Within the above limitations, the investigation fell into two parts, that is the effect on the coefficient of discharge, of

(1) the head H_s
(2) the ratio H_s/D

The diameters tested at Lehigh University were 0.375' diameter, 0.542' diameter, 0.708' diameter and 0.875' diameter. Results were also obtained for weirs of 1.13' diameter[5]; 1.14' diameter[6]; 1.66 diameter[7]; 2.0' diameter, 3.97' diameter and 5.96' diameter.[8]

The original data of all calibration and experimental tests carried out at Lehigh University are on file in Fritz Laboratory. A summary of all data is available.[4]

From all available experimental data, Fig. 17 was plotted. It shows the variation of $\frac{Q}{H_s^{1.5}D^{1.0}g^{0.5}}$ with the head H_s, for various constant values of the ratio H_s/D. For clarity, the ordinate has been plotted as $\frac{Q}{D^{2.5}g^{0.5}}$ (i.e. $\frac{Q}{H_s^{1.5}D^{1.0}g^{0.5}} \cdot \left(\frac{H_s}{D}\right)^{1.5}$), which, since curves are plotted for constant values of H_s/D, has only the effect of separating the curves.

The graph indicates that, when H_s is greater than approximately 0.10 ft., the value of $\frac{Q}{H_s^{1.5}D^{1.0}g^{0.5}}$ remains constant for each value of H_s/D up to $H_s/D \simeq 0.30$. That is, the relationship is independent of H_s (i.e. surface tension and/or viscosity) for heads greater than 0.10 ft., (corresponding to a Reynold's No. of 16,600 and a Weber No. of 125).

This provides more definite limits than Mr. Wagner's paper on the minimum size of weir which may be used so that "scale effects" can be eliminated. The inconsistencies in the discharge coefficient for low heads, mentioned by Mr. Wagner on p. 16, were not found in the Lehigh tests, probably due to the smaller time required to establish steady conditions. The tendency of the discharge coefficient to "increase as head decreased" was due to surface tension and/or viscosity. The separate effects of surface tension and viscosity can be determined only by tests with different fluids flowing over the weir. Until such tests are completed the present results apply only for water.

Fig. 18 shows the relationship between $\frac{Q}{H_s^{1.5}D^{1.0}g^{0.5}}$ and H_s/D for the range where the former is independent of H_s. On the log-log graph, this relationship plots as a straight line between the range of H_s/D from 0.03 to 0.30. The relationship is

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5. "Experiments on the Flow of Water Over Sharp-Edged Circular Weirs," by H.J.F. Gourley, Proc. of the Institution of Civil Engineers, Vol. 184 Part II, 1910-11, pg. 297.
6. "Determination of Undernappe Over a Sharp Crested Weir, Circular in Plan, with Radial Approach," by R. Dupont. Thesis submitted to Case School of Applied Science, Cleveland, Ohio, 1937.
7. Data received from Bureau of Reclamation, Denver, Colorado, of tests conducted by Mr. W.E. Wagner up till 29th Dec., 1951.
8. "Tests on Circular Weirs," by C. Camp and J.W. Howe, Civil Engineering, Vol. 9 No. 4, April 1939, pg. 247. (Original data kindly supplied by Professor J.W. Howe in January 1952—See Reference 4).

$$\frac{Q}{H_s^{1.50} D^{1.00} g^{0.50}} = 1.66 \left(\frac{H_s}{D} \right)^{-0.04}$$

$$\text{of} \quad Q = 1.66 \left(\frac{H_s}{D} \right)^{-0.04} \sqrt{g} \quad D \, H_s^{1.50}$$

$$\text{i.e.} \quad C = \frac{1.66}{\pi} \left(\frac{H_s}{D} \right)^{-0.04} \sqrt{g}$$

This is in very close agreement with the values given by Mr. Wagner (pg. 13 and Fig. 9) for the more Limited range of his tests.

The preceding equation can also be written

$$Q = 1.66 \sqrt{g} \quad D^{1.04} \quad H_s^{1.46}$$

In Fig. 19, the parameter $\frac{Q}{g^{0.5} H_s^{1.46} D^{1.04}}$ is plotted against the depth H_s. Once H_s exceeds approximately 0.10 ft., the value is constant at 1.66. For heads less than 0.10 ft., the available results for all the weirs fall on a smooth curve for heads as low as 0.02 ft. In addition to indicating that the effects of surface tension and/or viscosity are correctly represented by Reynolds and Weber Nos. involving the head only (as written), the graph also shows the value of the discharge coefficient for any head below 0.10 ft., when water is the fluid medium.

The point at which weir flow is "flooded out," the regime changing to flow through a reentrant tube, was found in the Lehigh experiments, to occur when H_s/D was approximately equal to 0.30, representing the upper limit, above which weir flow cannot exist. However, if a vortex is formed, this causes the flow to diverge from weir flow at an earlier point. This is probably the reason why Mr. Wagner and earlier investigators found the limit for "free" discharge to be lower than 0.30. The effect of vortex formation is to reduce the discharge for a given head,[9] as shown in Fig. 20, which is a discharge-head curve for an arbitrary one ft. diameter circular weir. If the vortex is eventually destroyed with the establishment of tube flow the vortex causes a gradual transition between the two types of flow. When a vortex was suppressed by means of simple radial baffles, then weir flow continued up to the upper limit at a vlue of $H_s/D \simeq 0.30$. Most morning-glory spillways have radial baffles to prevent the formation of vortexes.[3] No quantitative results have been obtained for the effect of the vortex in terms of the vortex strength.

The conclusions from the Lehigh tests may be summarized as follows:

For the flow of water over a sharp-edged circular weir, with the following limitations

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9. "How the Vortex Affects Orifice Discharge," by C.J. Posey and H. Hsu, Engineering News Record, March 9, 1950, pg. 30.

1. radial approach conditions (no vortex),
2. P greater than weir diameter,
3. aerated flow (i.e. undernappe at atmospheric pressure),

the relationship between the discharge, head over weir and weir diameter (up to point of "flood-out") can be expressed as

$$Q = K \left\{\frac{H_s}{D}\right\}^{-0.04} \sqrt{g} \quad D \; H_s^{1.50}$$

or

$$Q = K \sqrt{g} \quad D^{1.04} \quad H_s^{1.46}$$

The value of K is a function of the head over the weir H_s. If H_s exceeds 0.10 ft., K is constant, equal to 1.66. If H_s is less than 0.10 ft. the value of K may be obtained from Fig. 19 for heads down to 0.02 ft.

The upper limit at which weir flow changes in character to flow through a reentrant tube is given by $H_s/D = 0.30$; the formation of a vortex causes the flow to deviate from true weir flow at a lower value of this ratio.

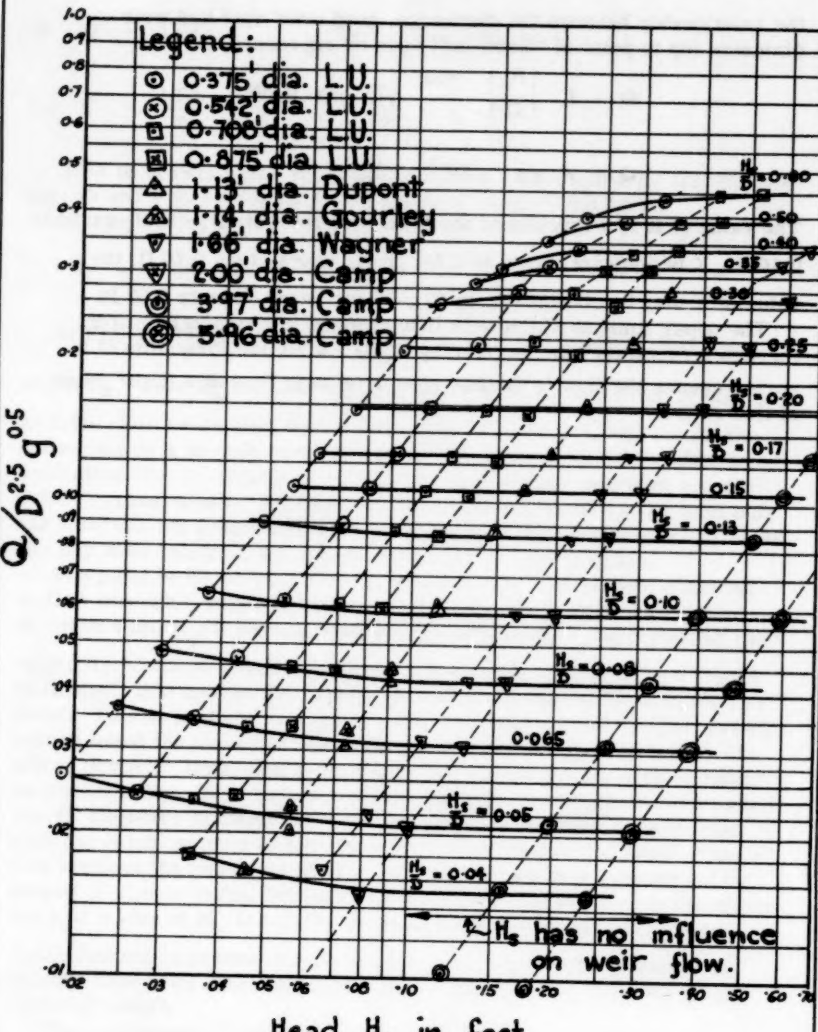

FIG. 17. GRAPH OF $\frac{Q}{D^{2.5} g^{0.5}}$ V. H_s FOR CONSTANT VALUES OF $\frac{H_s}{D}$.

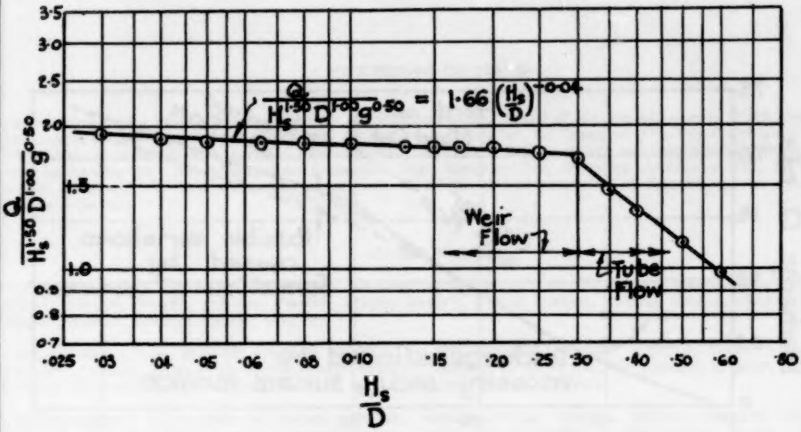

FIG. 18. GRAPH OF $\frac{Q}{H_s^{1.50} D^{1.00} g^{0.50}}$ V. $\frac{H_s}{D}$ FOR REGION WHERE H_s (I.E. N_R AND N_W) HAS NO INFLUENCE.

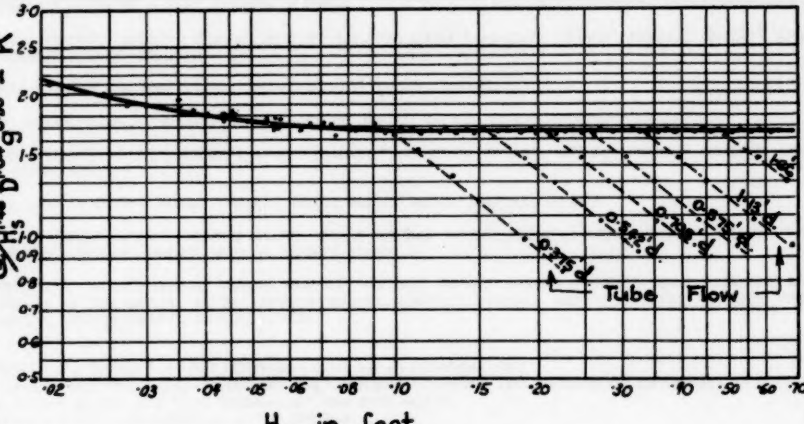

FIG. 19. GRAPH OF $\frac{Q}{H_s^{1.46} D^{1.04} g^{0.50}}$ V. H_s FOR WATER

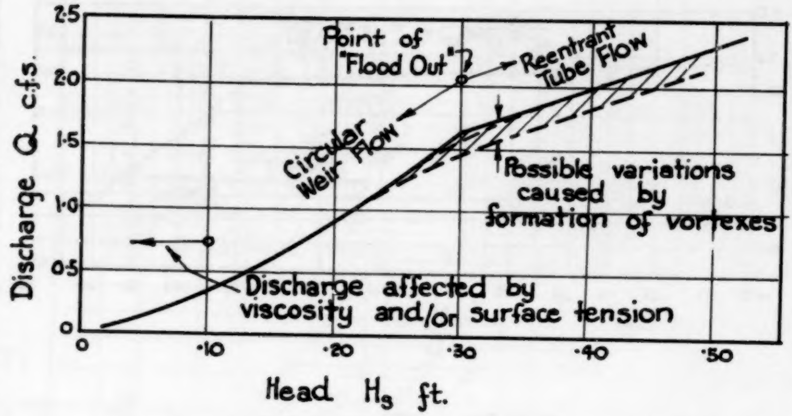

FIG. 20. HEAD-DISCHARGE RELATIONSHIP FOR FLOW OVER A 1'-0" DIA. CIRCULAR WEIR.

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 79 (1953)

SEPTEMBER: 260(AT), 261(EM), 262(SM), 263(ST), 264(WW), 265(ST), 266(ST), 267(SA), 268(CO), 269(CO), 270(CO), 271(SU), 272(SA), 273(PO), 274(HY), 275(WW), 276(HW), 277(SU), 278(SU), 279(SA), 280(IR), 281(EM), 282(SU), 283(SA), 284(SU), 285(CP), 286(EM), 287(EM), 288(SA), 289(CO).

OCTOBER:[b] 290(all Divs), 291(ST)[a], 292(EM)[a], 293(ST)[a], 294(PO)[a], 295(HY)[a], 296(EM)[a], 297(HY)[a], 298(ST)[a], 299(EM)[a], 300(EM)[a], 301(SA)[a], 302(SA)[a], 303(SA)[a], 304(CO)[a], 305(SU)[a], 306(ST)[a], 307(SA)[a], 308(PO)[a], 309(SA)[a], 310(SA)[a], 311(SM)[a], 312(SA)[a], 313(ST)[a], 314(SA)[a], 315(SM)[a], 316(AT), 317(AT), 318(WW), 319(IR), 320(HW).

NOVEMBER: 321(ST), 322(ST), 323(SM), 324(SM), 325(SM), 326(SM), 327(SM), 328(SM), 329(HW), 330(EM)[a], 331(EM)[a], 332(EM)[a], 333(EM)[c], 334(EM), 335(SA), 336(SA), 337(SA), 338(SA), 339(SA), 340(SA), 341(SA), 342(CO), 343(ST), 344(ST), 345(ST), 346(IR), 347(IR), 348(CO), 349(ST), 350(HW), 351(HW), 352(SA), 353(SU), 354(HY), 355(PO), 356(CO), 357(HW), 358(HY).

DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)[c], 368(WW)[c], 369(IR), 370(AT)[c], 371(SM)[c], 372(CO)[c], 373(ST)[c], 374(EM)[c], 375(EM), 376(EM), 377(SA)[c], 378(PO)[c].

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JANUARY: 379(SM)[c], 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)[c], 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)[c], 390(HY), 391(IR)[c], 392(SA), 393(SU), 394(AT), 395(SA)[c], 396(EM)[c], 397(ST)[c].

FEBRUARY: 398(IR)[d], 399(SA)[d], 400(CO)[d], 401(SM)[c], 402(AT)[d], 403(AT)[d], 404(IR)[d], 405(PO)[d], 406(AT)[d], 407(SU)[d], 408(SU)[d], 409(WW)[d], 410(AT)[d], 411(SA)[d], 412(PO)[d], 413(HY)[d].

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APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

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a. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
b. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.
c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.